

Worcester County Mathematics League

Freshman Meet 2

December 12, 2018

COACHES' COPY

ROUNDS, ANSWERS, AND SOLUTIONS

WORCESTER COUNTY MATHEMATICS LEAGUE



Freshman Meet 2 – December 12th, 2018

Answer Key

Round 1:

1. 144 (Hudson)
2. 122 (Bromfield)
3. 30 (Notre Dame)

Round 2:

1. 9 (Auburn)
2. 2131 (Millbury)
3. 7 (South)

Round 3:

1. 0 (Oxford)
2. 3.05 (St. Johns)
3. $\frac{4}{39}$ (AMSA)

Round 4:

1. 4 (Shepherd Hill)
2. 64 (Shepherd Hill)
3. 20 (Shepherd Hill)

Team Round:

1. {2, 3, 4, 5, 6, 7, 8, 9, 12, 14} (West Boylston)	5. 3 (Bartlett)
2. 19 (Westborough)	6. 6:00AM (Worcester Academy)
3. 8 Nickels and 14 Dimes (Shepherd Hill)	7. -65 (Bartlett)
4. $\frac{44}{5}$ or $8\frac{4}{5}$ or 8.8 (Algonquin)	8. $(x+3)(x+3)(x-3)$ (Bromfield)

WORCESTER COUNTY MATHEMATICS LEAGUE



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Round 1: Algebraic Word Problems

All answers must be in simplest exact form in the answer section.

NO CALCULATOR ALLOWED

1. Mr. Smith drove to New York in 3 hours. On his return trip traffic was heavy and the trip took 4 hours. If his speed going to New York was 12 miles per hour faster than his speed returning, how many miles was New York from his home?
2. Three angles of a triangle have the following characteristics: the measure of angle A is twenty degrees less than the measure of angle B. The measure of angle C is five degrees more than three times the measure of angle B. What is the measure of the largest angle of the triangle ABC?
3. Sue has 50-liters of a solution that is 10% salt. How much water must she evaporate to produce a solution that is 25% salt?

ANSWERS

(1 pt.) 1. _____

(2 pt.) 2. _____

(3 pt.) 3. _____



Freshman Meet 2 – SOLUTIONS

Round 1: Algebraic Word Problems

1. Mr. Smith drove to New York in 3 hours. On his return trip traffic was heavy and the trip took 4 hours. If his speed going to New York was 12 miles per hour faster than his speed returning, how many miles was New York from his home?

Solution:

$$3(x + 12) = 4x$$

$$3x + 36 = 4x$$

$$x = 36$$

$$4x = 144$$

Solution: 144

2. Three angles of a triangle have the following characteristics: the measure of angle A is twenty degrees less than the measure of angle B. The measure of angle C is five degrees more than three times the measure of angle B. What is the measure of the largest angle of the triangle ABC?

Solution:

$$\text{Angle A} = x - 20$$

$$\text{Angle B} = x$$

$$\text{Angle C} = 3x + 5$$

$$(x - 20) + x + (3x + 5) = 180$$

$$5x - 15 = 180$$

$$5x = 195$$

$$x = 39$$

Angle A = $x - 20$	19 Degrees
Angle B = x	39 Degrees
Angle C = $3x + 5$	122 Degrees

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Solution: 122

3. Sue has 50-liters of a solution that is 10% salt. How much water must she evaporate to produce a solution that is 25% salt?

Solution:

	Amount	% Salt	Amount Salt
Original Solution	50L	10%	.1(50)
Water	x	-	-
New Solution	50-x	25%	0.25(50-x)

$$0.25(50 - x) = 0.1(50)$$

$$12.5 - 0.25x = 5$$

$$-0.25x = -7.5$$

$$x = 30$$

Solution: 30

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Round 2: Number Theory

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. The integer 100 is divisible with no remainder by how many positive integers?

2. What is the value of 421 base 6 in base 4?

3. Given: 24, 40, 56 and 36
What is the greatest prime factor of their LCM?

ANSWERS

(1 pt.) 1. _____

(2 pt.) 2. _____

(3 pt.) 3. _____



Freshman Meet 2 – SOLUTIONS

Round 2: Number Theory

1. The integer 100 is divisible with no remainder by how many positive integers?

Solution:

The following are the numbers:

1, 2, 4, 5, 10, 20, 25, 50, 100

Solution: 9

2. What is the value of 421 base 6 in base 4?

Solution:

Change 421 base 6 to base 10

$$1 * 1 = 1$$

$$2 * 6 = 12$$

$$4 * 36 = \underline{144}$$

157 Base 10

Change 157 base 10 to base 4

$$\text{Start } \frac{157}{4} = 39 R 1 \text{ then } \frac{39}{4} = 9 R 3$$

$$\text{then } \frac{9}{4} = 2 R 1 \text{ and } \frac{2}{4} = 0 R 2$$

Starting with Most significant bit

Solution: 2131



3. Given: 24, 40, 56 and 36
What is the greatest prime factor of their LCM?

Solution:

$$24 = 2 * 2 * 2 * 3$$

$$40 = 2 * 2 * 2 * 5$$

$$56 = 2 * 2 * 2 * 7$$

$$36 = 2 * 2 * 3 * 3$$

Thus

$$LCM = 2 * 2 * 2 * 3 * 3 * 5 * 7$$

Solution: 7

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Freshman Meet 2 – December 12th 2018

**Round 3: Operations on Numerical Fractions, Decimals, Percents, and Percentage
Word Problems**

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. Find the value of the expression:

$$0.03(6) + 5.2(0.003) - 0.04(4.89)$$

2. A shirt originally priced at \$13.00 is discounted 15% by the store owner. What profit (in terms of dollars) does the store owner make if the shirt had cost him \$8.00?

3. A, B, C, D and E are consecutive points on a line. If $AB/BC = 1/3$, $BC/CD = 1/4$, and $CD/DE = 1/2$, what is AC/BE ?

ANSWERS

(1 pt.) 1. _____

(2 pt.) 2. _____

(3 pt.) 3. _____



Freshman Meet 2 – SOLUTIONS

**Round 3: Operations on Numerical Fractions, Decimals, Percents, and Percentage
Word Problems**

1. Find the value of:

$$0.03(6) + 5.2(0.003) - 0.04(4.89)$$

Solution:

$$0.03(6) + 5.2(0.003) - 0.04(4.89)$$

$$0.18 + 0.0156 - 0.1956$$

$$0.1956 - 0.1956$$

Solution: 0

2. A shirt originally priced at \$13.00 is discounted 15% by the store owner. What profit (in terms of dollars) does the store owner make if the shirt had cost him \$8.00?

Solution:

$$0.15 * 13 = 1.95$$

Therefore

Selling price is \$11.05

Less cost of \$8.00

Profit = \$3.05

Solution: 3.05

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3. A, B, C, D and E are consecutive points on a line. If $AB/BC = 1/3$, $BC/CD = 1/4$, and $CD/DE = 1/2$, what is AC/BE ?

Solution:

$$AC = AB + BC \text{ and } BE = BC + CD + DE$$

$$\text{Then } \frac{AC}{BE} = \frac{AB + BC}{BC + CD + DE} = \frac{1 + \frac{AB}{BC}}{1 + \frac{CD}{BC} + \frac{DE}{BC}}$$

$$\text{Now } \frac{AB}{BC} = \frac{1}{3}$$

$$\frac{CD}{BC} = \frac{1}{\frac{BC}{CD}} = \frac{1}{\frac{1}{4}} = 4$$

$$\frac{DE}{BC} = \frac{1}{\frac{BC}{DE}} = \frac{1}{\left(\frac{BC}{CD}\right)\left(\frac{CD}{DE}\right)} = \frac{1}{\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)} = 8$$

$$\text{So } \frac{AC}{BE} = \frac{1 + \frac{1}{3}}{1 + 4 + 8} = \frac{4}{39}$$

Solution: 4/39

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Round 4: Set Theory

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. $U = \{1, 2, 3, 4, \dots, 12\}$
 $A = \{1, 5, 7, 9\}$
 $B = \{4, 5, 6, 7, 8\}$
 $C = \{3, 4, 11, 12\}$

Evaluate:

$$(A \cup B) \cap C$$

2. If set $A = \{4, 5, 6, 7, 8, 9\}$; How many subsets can be made from set A?
3. In a survey of 100 students, the number of students studying various languages were found to be: Spanish 28; German 30; French 42; Spanish & German 8; Spanish & French 10; German & French 5; all three languages 3. How many students were studying no language?

ANSWERS

(1 pt.) 1. _____

(2 pt.) 2. _____

(3 pt.) 3. _____



Freshman Meet 2 – SOLUTIONS

Round 4: Set Theory

1. $U = \{1, 2, 3, 4, \dots, 12\}$
 $A = \{1, 5, 7, 9\}$
 $B = \{4, 5, 6, 7, 8\}$
 $C = \{3, 4, 11, 12\}$

Evaluate:

$$(A \cup B) \cap C$$

Solution:

$$(A \cup B) = \{1, 4, 5, 6, 7, 8, 9\}$$

$$(A \cup B) \cap C = 4$$

Solution: 4

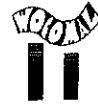
2. If set $A = \{4, 5, 6, 7, 8, 9\}$; How many subsets can be made from set A?

Solution:

Using 2^N where N is the total of elements in the set.

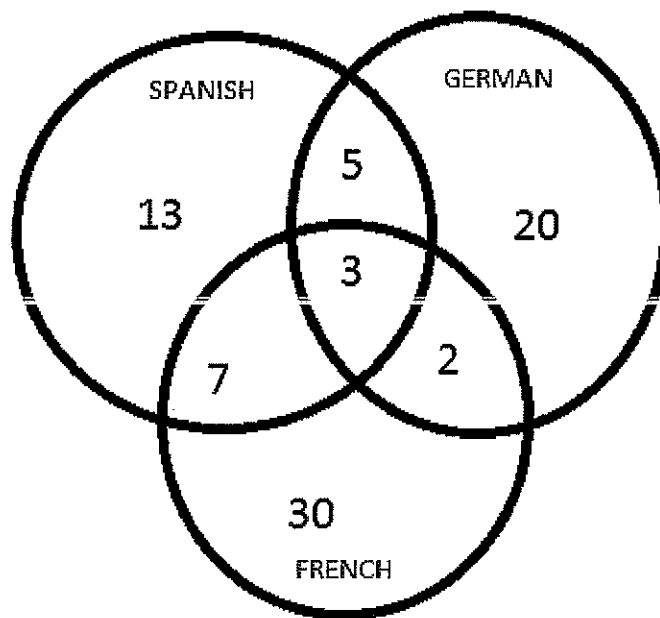
Therefore: set A has 6 elements so 2^6 subsets or 64 subsets

Solution: 64



3. In a survey of 100 students, the numbers of students studying various languages were found to be: Spanish 28; German 30; French 42; Spanish & German 8; Spanish & French 10; German & French 5; all three languages 3. How many students were studying no language?

Solution:



Total Students 100

Total Students taking Language 80

Students not taking Language 20

Solution: 20

WORCESTER COUNTY MATHEMATICS LEAGUE



Freshman Meet 2 – December 12th 2018

TEAM ROUND

All answers must be in simplest exact form in the answer section. (3 points each)

NO CALCULATOR ALLOWED

1. Solve for x:

$$x = \frac{(A \cup B) \cup (B \cap C)}{}$$

Where:

$$U = \{1, 2, 3, 4, \dots, 14\}$$

$$A = \{2, 4, 6, 8, 10, 12, 14\}$$

$$B = \{1, 3, 5, 7, 9, 10\}$$

$$C = \{1, 2, 6, 10, 14\}$$

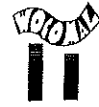
2. How many prime numbers are less than 70?
3. A coin bank contains \$1.80 in nickels and dimes; in all there are 22 coins in the bank. Find both the number of nickels and the number of dimes in the bank.
4. Evaluate: $\frac{y+5}{x+2} + \frac{3y+2}{1-2x}$ for: $x = \frac{1}{4}$ $y = 0.4$
5. How much driveway sealer in gallons will you need to make 2 applications on a driveway measuring 36 feet x 15 feet, if 1 gallon of sealer covers 40 square yards?
6. If at a certain time it is 8:00PM, what time will it be 11,994,994 hours later? Be sure to include AM or PM.
7. Evaluate the following expression for $y = -4$ and $z = -2$.

$$3(4z + 6) + 5[7 + 3(2y + 4) + 32] + y$$

8. Factor completely:

$$(x^2 - 9)(x+2) + (x+3)(x-2) - (x+3)$$

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TEAM ROUND ANSWER SHEET

1. _____

2. _____

3. Nickels = _____ Dimes = _____

4. _____

5. _____

6. _____

7. _____

8. _____

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TEAM ROUND - SOLUTIONS

1. Solve for x:

$$x = \overline{(A \cup B) \cup (B \cap C)}$$

Where:

$$U = \{1, 2, 3, 4, \dots, 14\}$$

$$A = \{2, 4, 6, 8, 10, 12, 14\}$$

$$B = \{1, 3, 5, 7, 9, 10\}$$

$$C = \{1, 2, 6, 10, 14\}$$

Solution:

$$\overline{(A \cup B)} = \{11, 13\}$$

$$(B \cap C) = \{1, 10\}$$

$$\overline{(A \cup B) \cup (B \cap C)} = \{1, 10, 11, 13\}$$

$$\overline{\overline{(A \cup B) \cup (B \cap C)}} = \{2, 3, 4, 5, 6, 7, 8, 9, 12, 14\}$$

$$\text{Solution: } \{2, 3, 4, 5, 6, 7, 8, 9, 12, 14\}$$

2. How many prime numbers are less than 70?

Solution:

Prime numbers less than 70: $\{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67\}$

Solution: 19



3. A coin bank contains \$1.80 in nickels and dimes; in all there are 22 coins in the bank. Find the number of nickels and dimes in the bank.

Solution:

Let x = number of nickels

Let $22-x$ = number of dimes

$$5x + 10(22 - x) = 180$$

$$5x + 220 - 10x = 180$$

$$-5x = -40$$

$$x = 8$$

Solution: 8 Nickels, 14 Dimes

4. Evaluate: $\frac{y+5}{x+2} + \frac{3y+2}{1-2x}$ for: $x = \frac{1}{4}$ $y = 0.4$

Solution:

$$\frac{5.4}{2\frac{1}{4}} + \frac{1.2 + 2}{1 - \frac{1}{2}}$$

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$$\frac{5\frac{2}{5}}{2\frac{1}{4}} + \frac{3\frac{1}{5}}{\frac{1}{2}}$$
$$\left(\frac{27}{5}\right)\left(\frac{4}{9}\right) + \left(\frac{16}{5}\right)\left(\frac{2}{1}\right)$$

$$\frac{12}{5} + \frac{32}{5} = \frac{44}{5}$$

Solution: $44/5$ or $8\frac{4}{5}$ or 8.8

5. How much driveway sealer in gallons will you need to make 2 applications on a driveway measuring 36 feet x 15 feet, if 1 gallon of sealer covers 40 square yards?

Solution:

$$36 * 15 = 540 \text{ square feet}$$

Convert 540 square feet by dividing by 9 = 60 Square Yards

$$60 \text{ square yards} * 2 \text{ applications} = 120 \text{ total square yards}$$

$$120/40 = 3$$

Solution: 3

6. If at a certain time it is 8PM, what time will it be 11,994,994 hours later? Be sure to include AM or PM!

Solution:

Perform division by 24, the remainder will be added to 8:00PM.

$$11994994/24 = 499791 \text{ with a remainder of } 10$$

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Solution: 6:00AM

7. Evaluate the following expression for $y = -4$ and $z = -2$.

$$3(4z + 6) + 5[7 + 3(2y + 4) + 3z] + y$$

Solution:

$$3(4z + 6) + 5[7 + 3(2y + 4) + 3z] + y$$

$$12z + 18 + 5[7 + 6y + 12 + 3z] + y$$

$$12z + 18 + 5[6y + 3z + 19] + y$$

$$12z + 18 + 30y + 15z + 95 + y$$

$$31y + 27z + 113$$

$$31(-4) + 27(-2) + 113$$

$$-124 - 54 + 113$$

$$-65$$

Solution: -65

8. Factor Completely:

$$(x^2 - 9)(x+2) + (x+3)(x-2) - (x+3)$$

Solution:

$$(x+3)[(x-3)(x+2) + (x-2) - 1]$$

$$(x+3)[x^2 - x - 6 + x - 2 - 1]$$

$$(x+3)(x^2 - 9)$$

$$(x+3)(x+3)(x-3)$$

Solution: $(x+3)(x+3)(x-3)$